1. A control engineer decides on the following specifications for \( G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \) (30)

(a) Peak time less than 0.2\( \pi \) seconds
(b) Settling time (2\%) less than 8 seconds
(c) Maximum overshoot less than \( e^{-\frac{\pi}{\sqrt{3}}} \).

Shade the region on the complex plane (\( Re - Im \) axes) within which the roots of the characteristic equation should lie for all these conditions to be satisfied.

2. The transfer function of the linear plant is given by \( G(s) = \frac{1}{s^2 + s+1} \). You are required to design a PI controller \( G_c(s) = k + \frac{k_i}{s} \) by properly selecting \( k \) and \( k_i \).

(a) First, determine the range of values for \( (k, k_i) \) for which the closed loop system is stable. (Draw the region of values in a coordinate plane \( k, k_i \) for which the system is stable.) (20)
(b) The steady state error (\( \lim_{t \to \infty} e(t) \)) when the reference signal is a unit ramp \( u_r(t) = t \) for \( t > 0 \), should not exceed 0.1. Determine the range of values for \( (k, k_i) \) that ensure stability and meet this constraint as well. (20)
(c) Large values for \( k \) and \( k_i \) may be undesirable. What is the minimal values for \( (k, k_i) \) that meet the constraint. (10)

3. Sketch the root loci for the system in the figure. (70)

4. Consider the Bode plot of a minimum phase system shown in the figure:

(a) How many poles and zeros does this system have? Explain. (10)
(b) Approximate the transfer function. (The DC gain is \(-20\text{dB}\) and the corner frequencies are located at 1, 10, 100 or 1000. Assume the damping ratio is 0.02.) (10)
(c) Find the steady state output for this system when the input is \( \sin 0.1t + 10\sin 2t \). (20)
5. Consider a system \( G(s) = \frac{\frac{1}{10}s + 1}{s^2 + 0.2s + 1} \).

(a) Draw the Bode plot. (You need to denote the corner frequencies and the DC gain.) (30)

(b) Sketch the Polar(Nyquist) plot. (You need to denote where the locus starts at \( \omega = 0 \) and the direction which it terminate with \( \omega \to \infty \)) (30)

(c) Find the phase margin and gain margin with the assumption that \( G(s) \) is an open-loop transfer function of the feedback system. (30)

6. Consider the Nyquist plot of a open-loop system \( G(s) \) shown in the figure. It is known that \( G(s) \) has two unstable poles. Is the closed loop system shown with unity feedback stable? Explain with the Nyquist stability criterion. (40)

7. Design a lead controller \( G_c(s) = \frac{Ts + 1}{s + \frac{1}{T}} \) for the plant \( G(s) = \frac{1}{s} \) such that the phase margin is \( 45^\circ \). The bode plot of \( G(s) \) is given below. (Observe that the phase plot is constant and therefore you do not need to add any extra phase to correct the required PM.) (80)